Error analysis of divergence and vorticity from aircraft-measured winds in a cloud-capped boundary layer

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Introduction

DYCOMS-II winds and synthetic wind fields are used to investigate errors in estimates of divergence and vorticity from airborne measurements of winds. Multidimensional linear regression is applied to estimate divergence and vorticity as the best fit of a linear wind model to the measured wind. Ensembles of synthetic wind fields are constructed to study the effect that turbulent wind fluctuations have on the estimates of the mean wind derivatives. The equations for calculating winds from the measured air velocity with respect to the airplane, velocity of the airplane with respect to the earth, and attitude angles are analyzed to study the effects of errors in flow and attitude angles on the wind field. DYCOMS-II winds are modified by changing the airflow and attitude angles to illustrate the error propagation. The attack and pitch angles are modified in an attempt to align the axes of instruments for measuring airflow and airplane attitude angles, as is assumed to be in the Air Motion Equations, and to re-evaluate divergence.

Wind data and estimates of divergence and vorticity

Data from seven out of nine DYCOMS-II research flights (RF) are used in this study to estimate divergence and vorticity via a circular flight pattern. During each RF there were up to eight 30 min flight legs at for levels within the PBL with two legs at each level. The circles were flown clockwise (CW) and counterclockwise (CCW) to cancel errors modulated by the aircraft orientation. As Fig. 1 shows, the air-relative flight path is very close to a circle. This circular flight path has an advantage that in horizontally homogeneous conditions the mean wind observed during one flight leg is a linear combination of time and *sin* and *cos* functions of the airplane azimuth (ψ_a). Coefficients in this linear wind model represent divergence and vorticity of horizontal wind and can be estimated by least-squares fit:

$$u = u_0 + \frac{\partial u}{\partial x} R \sin\psi_a + \frac{\partial u}{\partial y} R\cos\psi_a + \frac{\partial u}{\partial t}\delta t$$

$$v = v_0 + \frac{\partial v}{\partial x} R \sin\psi_a + \frac{\partial v}{\partial y} R\cos\psi_a + \frac{\partial v}{\partial t}\delta t$$

$$D = \underline{D_x} + \underline{D_y} = \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y}$$

$$\zeta = \zeta_x + \zeta_y = \frac{\partial v}{\partial x} - \frac{\partial u}{\partial y}$$

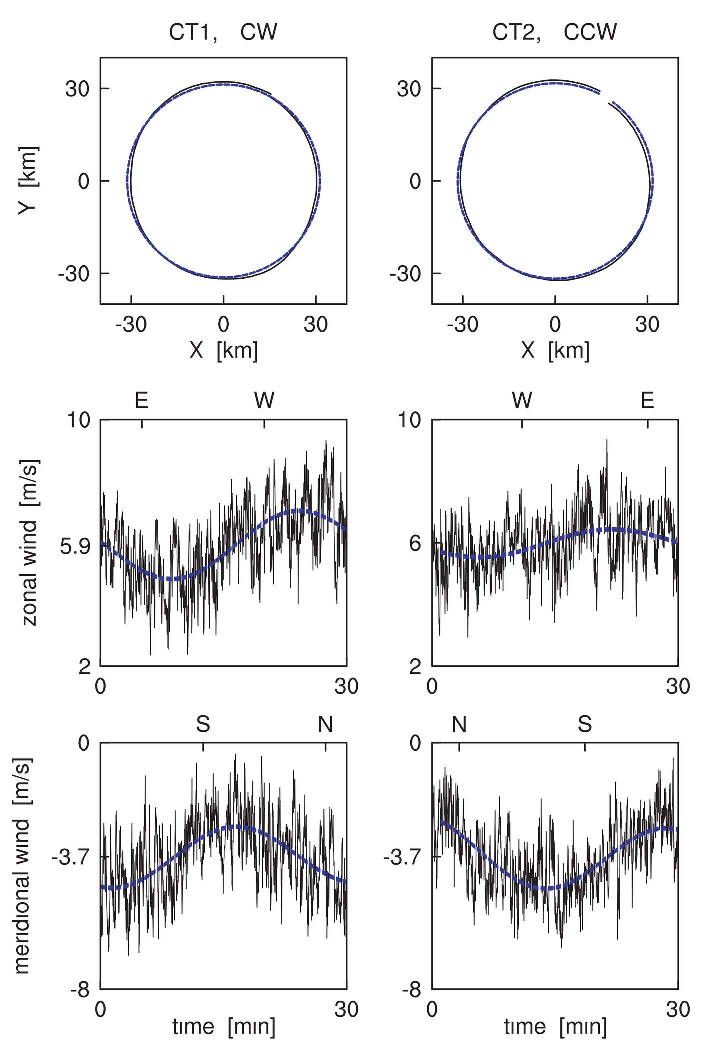

Figure 1: Air-relative flight path and measured wind field for two flight legs just below the cloud top during RF07 (solid black line). Best fit of circular path and linear wind field (dashed blue line).

Initial results

The regression technique applied to DYCOMS-II data initially yielded puzzling results: both divergence (Fig. 2) and vorticity were negative. The error analysis based on synthetic wind fields, chosen to mimic the mean turbulent characteristics of DYCOMS-II winds, revealed that turbulent fluctuations are a significant source of error (~6 10^{-6} s^{-1} per circle), but not big enough to change the sign of the divergence (Fig. 2, right). Fig. 2 (left) shows that the leg-mean divergence estimates are in agreement with the measured wind. This led to the conclusion that an unforeseen error existed and motivated a closer analysis of the wind measurements.

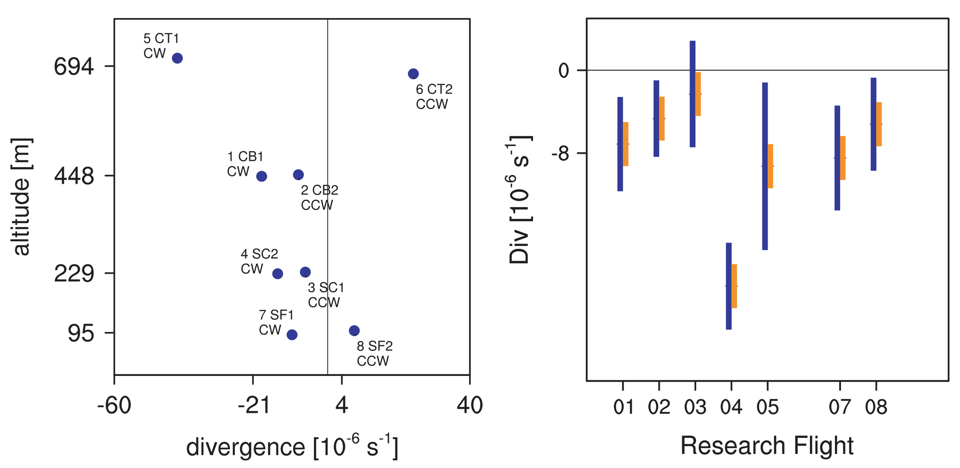

Figure 2: Leg-mean divergence estimates for RF07 (left panel). Flight-mean divergence estimates for all DYCOMS-II flights (right panel). Blue error bars are observed sampling error. Orange error bars are expected sampling error due to the turbulent variations in the wind field.

Wind measurement

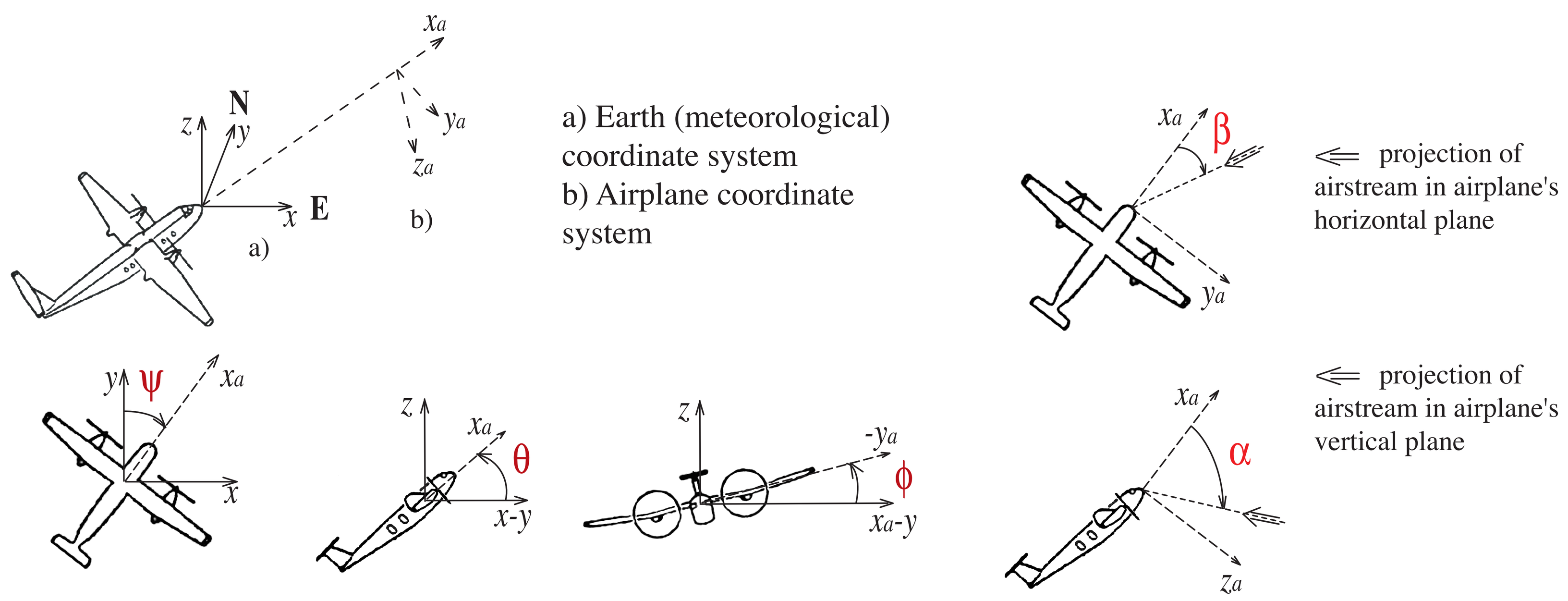

Figure 3: Coordinate systems, airplane attitude angles and airplane-relative airflow angles as in the equations for calculating the air velocity components (Lenschow equations).

Airplane measurements of the wind are based on measurement of ariplane attitude angles (true heading, ψ, pitch, θ, and roll, φ), airplane velocity (u_p,v_p,w_p), airplane-relative airflow angles (attack angle, α, and sideslip angle, β) and true airspeed (U_a) (Fig. 3). The wind is then calculated as the vector sum of airplane velocity and airplane-relative airflow velocity in the Earth coordinate system. Equations used for the wind calculations are:

$$u = u_p + \cos\psi[-U_a D_a^{-1}(-\tan\alpha\sin\phi + \tan\beta\cos\phi) + L\dot{\psi}\cos\theta] + \sin\psi\{\cos\theta(-U_a D_a^{-1}) + \sin\theta[-U_a D_a^{-1}(\tan\alpha\cos\phi + \tan\beta\sin\phi) - L\dot{\theta}]\}$$

$$v = v_p + \sin\psi[-U_a D_a^{-1}(\tan\alpha\sin\phi - \tan\beta\cos\phi) - L\dot{\psi}\cos\theta] + \cos\psi\{\cos\theta(-U_a D_a^{-1}) + \sin\theta[-U_a D_a^{-1}(\tan\alpha\cos\phi + \tan\beta\sin\phi) - L\dot{\theta}]\}$$

$$w = w_p + \sin\theta(-U_a D_a^{-1}) - \cos\theta[-U_a D_a^{-1}(\tan\alpha\cos\phi + \tan\beta\sin\phi) - L\dot{\theta}]$$

$$D_a = \sqrt{1 + \tan^2\alpha + \tan^2\beta}$$

Offsets in the airflow angles

Figure 4: The effect that an offset in the sideslip angle has on measured wind when airplane flies horizontally through motionless air.

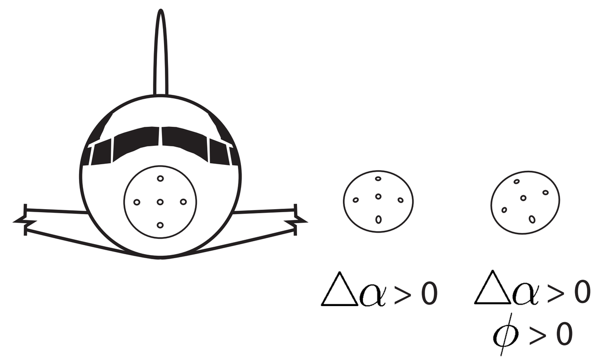

Figure 5: The effect that an offset in the attack angle has on measured wind when airplane flies a) horizontally and b) with some roll through motionless air.

$$u_{B\beta} \approx -U_a D_a^{-1}\cos^{-2}\beta\left(\pm\cos\psi_a(-D_a^{-2}\tan\beta\cos\theta) \mp \sin\psi_a(\cos\phi)\right)\triangle\beta$$

$$v_{B\beta} \approx -U_a D_a^{-1}\cos^{-2}\beta\left(\pm\cos\psi_a(-\cos\phi) \mp \sin\psi_a(-D_a^{-2}\tan\beta\cos\theta)\right)\triangle\beta$$

$$u_{B\alpha} \approx -U_a D_a^{-1}\cos^{-2}\alpha\left(\pm\cos\psi_a\sin(\theta-\alpha) \mp \sin\psi_a(\underline{-\sin\phi})\right)\triangle\alpha$$

$$v_{B\alpha} \approx -U_a D_a^{-1}\cos^{-2}\alpha\left(\pm\cos\psi_a(\underline{\sin\phi}) \mp \sin\psi_a\sin(\theta-\alpha)\right)\triangle\alpha$$

Based on the error analysis of the air-motion equations we found that the greatest source of uncertainty is associated with offsets in measurements of the attack and sideslip angles (Δα and Δβ). The manner in which these offsets bias the wind measurement is illustrated in Fig. 4 and 5: for straight and level legs, Δβ and Δα affect the lateral wind and the vertical velocity, respectively, but for flight legs that involve roll, a spurious vertical component due to Δα projects onto lateral wind as well. As the analytical expressions for the wind bias for a circular flight path show, when flying a pair of circles in opposite directions the effects of Δβ on both divergence and vorticity, as well as the effect of Δα on the vorticity, cancel out, while the effects of Δα on the divergence for both circles add up. The reason for this is that the change of flight direction is accompanied by a change in the direction of rolling, and therefore the terms that are proportional to the roll angle cannot be canceled. Because in the above expressions the only wind bias terms that are proportional to the *sin* of roll angle are those that affect the divergence, when flying in circles the divergence is the only one affected by the offsets in the airflow angles, and specifically by the offset in the attack angle.

Sensitivity

	$\triangle\psi$	$\triangle\theta$	$\triangle\phi$	$\triangle\beta$	$\triangle\alpha$	$\triangle U_a$
$\triangle u$ [m s^{-1}]	0.0	0.0	0.0	0.0	0.0	0.0
$\triangle v$ [m s^{-1}]	0.0	0.0	0.0	0.0	0.0	0.0
$\triangle w$ [m s^{-1}]	0.0	-1.9	0.0	0.0	1.9	0.0
$\triangle D$ [10^{-6} s^{-1}]	0.	0.	0.	0.	-5.	0.
$\triangle\zeta$ [10^{-6} s^{-1}]	0.	0.	0.	0.	0.	0.

Table 1: Sensitivity of mean air velocity components and wind derivatives on the offsets in the airplane attitude and airplane-relative airflow angles. ($\Delta\psi = \Delta\theta = \Delta\phi = \Delta\beta = \Delta\alpha = 1^o$ and ΔU_a = 1 m s^{-1})

As the analysis of the air-motion equations suggested a robustness of the airborne measurements of mean wind and vorticity along the circular flight path, we modified the DYCOMS-II winds by changing the airflow and airplane attitude angles by 1^o and true airspeed by 1 m s^{-1} to illustrate the error propagation. As Table 1 suggests the result is very encouraging. Flying along the pairs of circles in opposite directions, gives the flexibility to the instruments to have offsets, but still measure the mean atmospheric motion and vorticity with no error. Because the current calibration technique equalizes the pitch and attack angles when flying straight and level legs in conditions with no vertical motion, the effects of pitch and attack angles on vertical velocity that are of opposite sign are neutralized. Divergence requires another constraint on the measurement, but only on the accuracy of the attack angle measurement. The flexibility in measuring the sideslip angle offset comes from flying in opposite directions that cancels the error. To have the same flexibility in measuring the attack angle offset, aircraft would have to be symmetric about the horizontal plane and to fly upside down.

Zero mean pitch

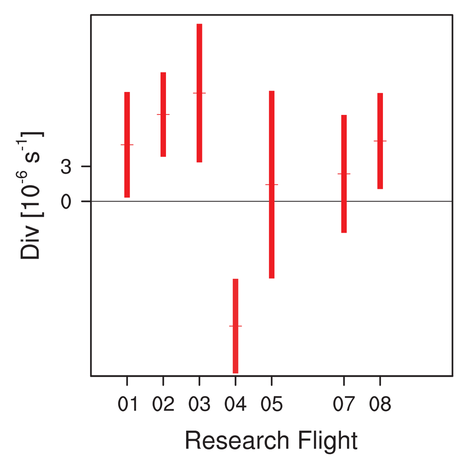

Figure 6: Re-evaluated flight-mean divergence estimates from modified wind data for all DYCOMS-II flights.

Due to current calibration techniques the offsets in the airflow angles can be interpreted as a misalignment in axes of instruments for measuring airplane-relative airflow velocity and aircraft attitude and velocity. In attempt to study the effect of this misalignment, which leads to the wind bias, the wind has been recalculated after a mean pitch angle has been subtracted from both attack and pitch angles. Fig. 6 shows that this re-evaluation of wind leads to the divergence estimates that appear more reasonable. However, the appropriateness of this change remains uncertain.

Conclusions

1) We have found that circles introduce a new error in the analysis of mean wind fields that has not been previously considered since the error does not occur on straight and level flight legs. The error arises because offsets in attack angle add a bias to the lateral wind component that is proportional to the roll angle. Since the roll angle changes sign when the circle is flown in opposite directions, the lateral wind component bias also changes sign, so that when circles are flown in opposite directions, the bias is not modulated by the change in rotation.

2) Previously, offsets in attack angle were dealt with by assuming zero mean vertical velocity over some reference flight track, then adding a mean offset to the attack angle to give a zero mean vertical velocity. This arbitrary procedure works for both mean winds and turbulence from straight and level flight tracks, but is not adequate when flying circles to measure divergence.

3) We are still working on a satisfactory resolution of this problem. One basic issue is that the pitch angle as measured by the inertial reference unit is not aligned with respect to the airplane axis that defines the reference system for measuring attack angle. We have not paid much attention to this in the past. Now we see that it is important to do so for divergence measurements from circular flight tracks.

4) It is likely that one of the results of this study will be to define procedures and techniques that need to be implemented in order to carry out successful measurements of divergence. It may still be possible to implement these procedures for DYCOMS analysis. We likely will include recommendations for new calibration techniques and more accurate air flow angle sensors.